Mathematics as Story

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Mathematical literature

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Our group came together with a common interest in mathematical literature. The issues we considered are summarized by the following questions: (1) What are the characteristics of existing mathematical literature? (2) How do we write a better mathematics story?
We expand on each of these questions below.

What are the characteristics of mathematical literature?

We started our deliberations by considering samples of mathematical literature. We flipped through books and chatted about the depth of the mathematics involved as well as the quality of the stories told.

We suggested five categories for categorizing mathematical literature: (1) A novel such as, *Jayden's Rescue* or *Uncle Petros and Golbach's Conjecture.* (2) A biography such as, *Mathematicians are People Too.* Typically biographical facts are presented as information in mathematics textbooks and teacher resources. These are often written in sidebars of textbook margins and contain sparse details. The connection of the person to the mathematics is weak. When mathematicians are presented in this manner, historical context and the social implications of mathematics is lost. The concern of making mathematics less accessible and tedious by recounting dry biographical facts was expressed by several of the symposium participants who suggested that we do not want to do to mathematics what historians have done to social studies. Emerging alternatives to this approach include stories about a mathematician's life, stories of the discipline of mathematics over time, stories of the development of mathematical concepts, and personal stories of encounters with mathematics. The essence of a good story can contribute to sense-making and enjoyment of mathematics as it engages the student. (3) A picture book such as, *Counting on Frank* or *Anno's Mysterious Multiplying Jar.* (4) A comix such as, *Comic-Strip Math.* (5) Poetry such as, *Marvelous Math: A Book of Poems.*

We also discussed what an annotated mathematical story might look like, where the annotations provide extensions to the mathematics discussed in the story. The annotations may appear in the margins of the story, as in *The Annotated Alice.* The story may also be presented electronically, with interactive/dynamic annotations. This discussion eventually led to a practical task we undertook,

namely, to consider how we might annotate a section of Lewis Carroll's story *A serpent with corners*. We will come back to this later on in this paper.

While considering examples of mathematical stories we discussed some concerns about the quality of some of the stories: (1) The mathematics appeared contrived or the mathematics seemed tangential to the story itself. (2) The 'pleasure' of the story did not arise from mathematical experiences. We also considered cases where the teacher 'found' mathematics in stories that were not explicitly mathematical. The story, or part of a story, served as a context for asking mathematical questions and exploring mathematical relationships not explicitly addressed in the story.

We suggested that students (and teachers) might play a variety of roles in classroom uses of mathematical literature: (1) Consuming literature. We wondered what types of mathematical literature appeals to students and to teachers. What might be the differences? What is the nature of the mathematics that emerges in each case? Are students more likely to be drawn to superficially appealing (non-mathematical) features of mathematical literature while teachers are more likely drawn to features that address mathematical ideas? What if the core of the story serves as 'sugarcoating' for mathematics. Is this desirable? Does it serve as a way into mathematics? Or, does it reinforce the view that mathematics is dry and needs

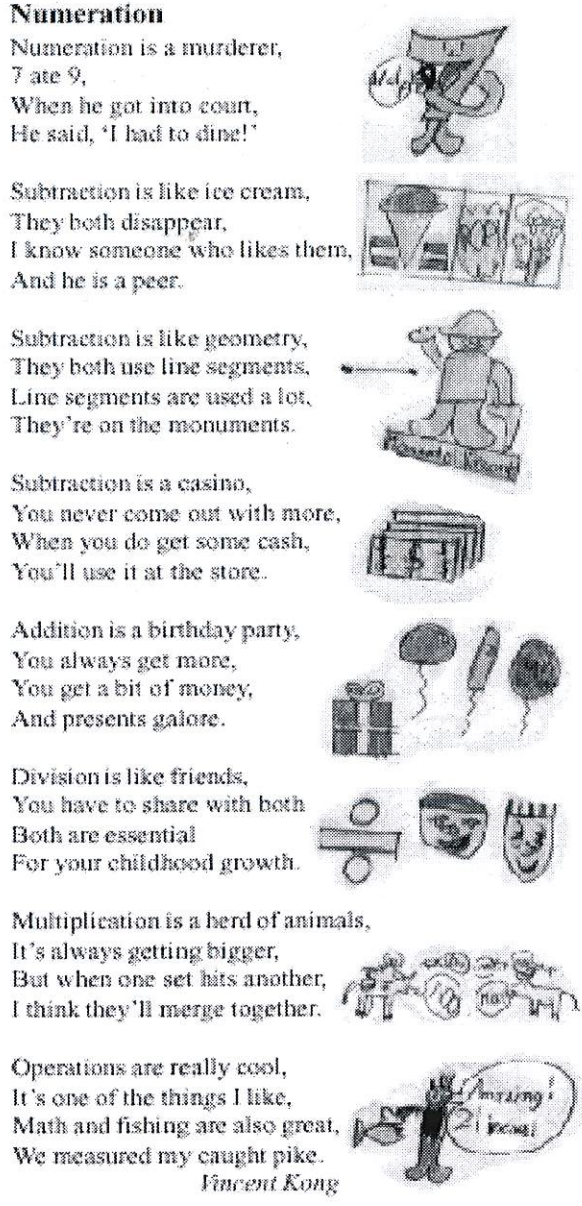

Figure 1. A student-authored poem.

more appealing, non-mathematical wrapping? (2) Creating literature. Here students become authors of mathematical stories, poems, comix, and so forth. An example of a student-authored poem is shown in Figure 1. (3) Extending literature. Here students extend the mathematics involved in a story, creating new contexts of constraints in which to investigate concepts. In each case, we wondered about the nature and degree of the mathematical attention brought to bear by students and by teachers.

How might we write a better mathematics story?

We suggest that one way to possibly improve mathematical literature, in the context of mathematics education, might be to provide annotations that lead the reader to a better understanding of the mathematics involved and offer opportunities to explore and extend the mathematics addressed in the story.

Consequently, as a culminating activity for our deliberations, we engaged in the practical task of annotating a mathematical story. We settled on Lewis Caroll's *A Serpent with Corners* (see Figure 2), due to its length (short enough given our time constraints) and due to its high mathematical potential.

As we read and discussed the story, we noticed that the flow of the story was interrupted at places. For example, the description of the rectangle as being 'oblong' caused us to question whether this referred to a rectangular shape with length greater than width, or whether is also referred to elliptical shapes. It seemed to us that such an interruption, such an ambiguity, was desirable, as it caused us to imagine alternatives and to wonder about a variety of contexts for the problem stated by Balbus in the story.

We suggest that the annotations might include: (1) Diagrams depicting mathematical situations. (2) Definitions of terms. (3) Mathematical extensions (new contexts or constraints)

We also imagined that the annotated story might be in electronic form, with the annotations allowing for interactive investigation. *Squeak* (a multimedia programming environment) was suggested as a development platform.

Involving students in annotating existing mathematical literature may be a way to integrate the three roles of students identified earlier, namely, consuming, extending, and authoring: Initially, students are consumers of the literature. They read the story before they consider any annotations. Then, through the annotations they create, they extend the story, especially its mathematical aspects. Finally, in annotating the story, students become authors of extensions to the story. In fact, some of the annotations may take the form of related stories, poems, and so forth.

"A friend of mine has a flower-garden – a very pretty one, though no great size –"

"How big is it?" said Hugh.

"That's what *you* have to find out!" Balbus gayly replied. "All *I* tell you is that it is oblong in shape – just half a yard longer than its width – and that a gravelwalk, one yard wide, begins at one corner and runs all around it."

"Joining onto itself?" said Hugh.

"*Not* joining onto itself, young man. Just before doing *that*, it turns a corner, and runs around the garden again, alongside of the first portion, and then inside that again, winding in and in, and each lap touching the last one, till it has used up the whole of the area."

"Like a serpent with corners?" said Lambert.

"Exactly so. And if you walk the whole length of it, to the last inch, keeping in the centre of the path, it's exactly two miles and half a furlong. Now, while you find out the length and breadth of the garden, I'll see if I can think out that sea-water puzzle."

"You said it was a flower-garden?" Hugh inquired as Balbus was leaving the room.

"I did," said Balbus.

"Where do the flowers grow?" said Hugh. But Balbus thought it best not to hear the question. He left the boys to their problem, and, in the silence of his own room, set himself to unravel Hugh's mechanical paradox.

Figure 2. *Excerpt from "A serpent with corners", from Lewis Carroll's* A Tangled Tale.