

The University of Western Ontario
Department of Mathematics

MATH 4155A/9055A - Calculus on Manifolds - Fall 2018

Time and Location: Tu 10:30–12:30 and Th 11:30–12:30 in room MC 107.

Instructor: Martin Pinsonnault

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Course website: Important announcements and documents will be posted on the OWL MATH 4155A/9055A website. Visit this page regularly for up to date information.

Prerequisites: CALC 2503A/B and MATH 3122A/B

Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.

Course Outline: The aim of this course is to provide an introduction to manifold theory for students who have no previous knowledge of the subject. We should cover the basics: review of vector calculus in \mathbf{R}^n from a modern point of view, manifolds (definition, examples, constructions), orientation, functions, partitions of unity, tangent bundle, cotangent bundle, vector fields, integral curves, differential forms, integration, manifolds with boundary, Stokes' theorem, submersions, immersions, embeddings, submanifolds, Sard's theorem, Whitney embedding theorem. If time permits, topics like de Rham cohomology or Lie groups may be introduced.

Textbook: The course will be based on

- Renteln, Paul, *Manifolds, Tensors and Forms*, CUP, 2013.
Covers the basics of multilinear algebra, differentiation and integration on manifolds, Lie groups and Lie algebras, homotopy and de Rham cohomology, homology, vector bundles, Riemannian and pseudo-Riemannian geometry, and degree theory. It also features over 250 detailed exercises.
- Tu, Loring W., *An Introduction to Manifolds*, Universitext, Springer, 2011.
This textbook is available electronically through UWO libraries. It covers slightly more material than Renteln's, in more depth.

References: I also encourage students to explore the subject with the help of some classical expositions. There is now a plethora of introductory books on manifolds and differential topology. Here is a list of some of my favorite references, from elementary to more advanced expositions:

- “Advanced Calculus” by Loomis & Sternberg
Freely available at http://www.math.harvard.edu/~shlomo/docs/Advanced_Calculus.pdf

- “Analysis on manifolds” by James R. Munkres
A very good classical exposition of differential and integral calculus in the language of differential forms. Contains examples and problems.
- “Calculus on manifolds” by Michael Spivak
Short and right to the point. Excellent for ”active” students that like to work out proofs by themselves and to sweat a bit more than the average fellow.
- “Introduction to Smooth Manifolds” , by John Lee
Covers almost exactly what I would like to focus on with many examples. Probably one of the best reference for this course.
- “Manifolds and Differential Geometry” by Jeffrey Lee
Similar to Jeffrey Lee’s book with additional topics like connections and Riemannian Geometry. Also a very good reference for this course.
- “Foundations of Differential Manifolds and Lie Groups” by Frank Warner
Neat, brief and concise, almost terse. Also more formal than the above references. Gives very few concrete examples, but covers sheaf cohomology and Hodge theory.
- “Differential Topology” by Morris Hirsch
In the spirit of Warner’s book. Focuses on foundational questions about spaces of functions on manifolds. Also gives a brief introduction to Morse theory and works out the classification of smooth surfaces.
- “Differential and Riemannian Manifolds” by Serge Lang
Also more advanced, it is one of the few textbooks that covers infinite dimensional (Banach) manifolds.

Note also that many lecture notes can be found on the web. Beware of mistakes and typos!

Evaluation: The evaluation will be based on

- 3 Homeworks: $\sim 30\%$
- Midterm: $\sim 30\%$
- Final Exam 40%.

The homeworks are an integral part of the course and special attention must be paid on redaction. They will be evaluated both on **correctness and clarity**.

Each student will give a presentation on a topic related to the course material. The topic should be chosen as early as possible. I will maintain a list of possible topics but you can propose any subject of particular interest/relevance for you. You will have to write a report of your presentation and to hand it in **at least a week before the talk** in order to get preliminary comments and to make corrections if necessary. The final version should be given **no later than a week after the talk**.

If you are unable to attend the midterm due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean’s office as soon as possible

and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor. For further information please see

http://www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_illness.pdf

Academic Offences: Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site: http://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_undergrad.pdf.

Accessibility: Please contact the course instructor if you require material in an alternate format or if you require any other arrangements to make this course more accessible to you. You may also wish to contact Services for Students with Disabilities (SSD) at 661-2111 ext. 82147 for any specific question regarding an accommodation.

Health and Wellness: As part of a successful student experience at Western, we encourage students to make their health and wellness a priority. Western provides several on campus health-related services to help you achieve optimum health and engage in healthy living while pursuing your degree. Students who are in emotional/mental distress should refer to Mental Health@Western <http://www.uwo.ca/uwocom/mentalhealth/> for a complete list of options about how to obtain help.