

The University of Western Ontario  
Department of Mathematics

## MATH 9302L - Riemann Surfaces - Summer 2022

**Instructor:** Martin Pinsonnault

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**Lectures:** Tuesday and Thursday, from 10:00 to 12:00, in room MC 108.

**Course's website:** Important announcements and documents will be posted on the MATH 9302L OWL website. Please visit this page regularly for up-to-date information on the course.

**Prerequisites:** Complex analysis and topology are essential. Familiarity with basic algebraic topology and calculus on manifolds are recommended, but we will review the necessary results as we go.

**Course Outline:** The aim of this course is to provide an introduction to Riemann surfaces. This subject lies at the intersection of complex analysis, algebraic geometry, and Riemannian geometry. As such, it can be presented in many different ways, each one focussing on different aspects of the theory.

Here is a tentative list of topics I plan to cover:

- Riemann surfaces and holomorphic maps between them, examples: algebraic curves, Riemann surfaces from analytic continuation, conformal structures.
- Covering spaces and monodromy, Riemann's existence theorem.
- Differential forms, de Rham and Dolbeault cohomology, Laplace operators, harmonic forms, The Dirichlet principle and Hodge theory à la Riemann.
- Elliptic integrals and functions, doubly periodic function, Weierstrass  $\wp$ -function, The field of meromorphic functions, Theta functions, classification of Riemann surfaces of genus one, the modular curve.
- The Riemann-Hurwitz formula, the degree-genus formula. Field of meromorphic functions, birational equivalence, connections with algebraic number theory. Hyperbolic surfaces, Gauss-Bonnet theorem.
- The main theorem for compact Riemann surfaces and its consequences: The Riemann-Roch formula, the uniformization theorem, automorphisms of Riemann surfaces, Weierstrass points.
- Divisors, line bundles, sheaf cohomology, Jacobi's inversion problem, Abel-Jacobi map, Jacobian of Riemann surfaces. Abelian varieties and abelian functions.
- Moduli spaces of Riemann surfaces, Beltrami differentials, compactification of moduli spaces.

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**References:** We will closely follow the following textbook:

- Simon Donaldson, *Riemann Surfaces*, Oxford Graduate Texts in Mathematics, vol. 22, Oxford University Press, 2011.

This textbook keeps an interesting balance between concrete motivational examples and abstract theory. It is also remarkable in the clarity of exposition while keeping the required prerequisites to a bare minimum. The approach is close to Riemann's original analytic approach based on potential theory and PDE's.

We will also borrow from other well-known references that are listed below. In order to get more insights, I encourage each student to use more than one reference. If you use other online sources, keep in mind that they may contain typos and other inaccuracies.

- O. Forster, *Lectures on Riemann surfaces*.
- H. M. Farkas, I. Kra, *Riemann Surfaces*.
- J. Jost, *Compact Riemann Surfaces*.

**Evaluation:** The evaluation will consist in

- 3 homeworks: 30%
- Presentation: 20%
- Written report: 20%
- Final exam: 30%

The homeworks are an integral part of the course and special attention must be paid on redaction. They will be evaluated on both correctness and clarity. Students are allowed to discuss assignment problems, but each student should write their solutions separately.

At the end of the course, everyone will give a presentation (about 50 minutes) on a subject related to the course material. The topic should be chosen as soon as possible. I will maintain a list of possible topics but you can propose any subject of particular interest/relevance for you. The presentations cannot use slides, must include significant results, and proofs of some of them. You will have to write a report prior to your presentation, written in LaTeX, and to hand it in (on paper or by email) **at least one week before the talk** in order to get preliminary comments and to make corrections if necessary. The final version should be given or emailed to me before the end of the term.

**Academic Integrity:** Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at this website: [http://www.uwo.ca/univsec/pdf/academic\\_policies/appeals/scholastic\\_discipline\\_grad.pdf](http://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_grad.pdf). Examples include copying solutions from other students or from other sources (e.g. online sources, textbooks), uploading questions or solutions to a web site; and accessing a web site used for providing solutions (even if you do not use any solutions on the site). The penalty for an

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academic offence involving a final exam will be a failing grade in the course. The penalty for each academic offence involving homework will be a grade of 0 on the homework set as well as a penalty of negative 5% on the overall course grade. In some cases, the penalty can include expulsion from the program. All academic offences are added to your student record.

**Attendance:** Our class is small and someone's absence can greatly impact the rest of the class. Therefore you are expected to attend class or to let me know in advance when you are unable to attend.

**Online access required:** We will be using Gradescope for homeworks. By taking this course, you are consenting to the use of this software. Completion of this course will require you to have a laptop or computer, a reliable internet connection, a working microphone, a working webcam, and a device for scanning (either a scanner or an app that can be used in conjunction with your device's camera).

**UWO e-mail:** The centrally administered e-mail account provided to students will be considered the individual's official university e-mail address. It is the responsibility of the account holder to ensure that e-mail received from the University at his/her official university address is attended to in a timely manner.

**Medical Accommodation:** A student requiring academic accommodation due to illness should bring a Student Medical Certificate with them when visiting an off-campus medical facility and use a Record Release Form for visits to Student Health Services. If homework is missed and sufficient documentation is provided, the homework can be handed in later. Failure to follow these rules may result in a grade of zero.

**Support Services:** Learning-skills counsellors at the Student Development Centre are ready to help you improve your learning skills. Students who are in emotional/mental distress should refer to Mental Health@Western for a complete list of options about how to obtain help. Additional student-run support services are offered by the USC. The website for Registrarial Services is <http://www.registrar.uwo.ca>.

**Student Accessibility Services:** Please contact the course instructor if you require material in an alternate format or if you require any other arrangements to make this course more accessible to you. You may also wish to contact Student Accessibility Services (formerly Services for Students with Disabilities, SSD) at 519-661-2111 x82147 for any specific question regarding an accommodation.

Western is committed to achieving barrier-free accessibility for all its members, including graduate students. As part of this commitment, Western provides a variety of services devoted to promoting, advocating, and accommodating persons with disabilities in their respective graduate program.

Graduate students with disabilities (for example, chronic illnesses, mental health conditions, mobility impairments) are encouraged to register with Student Accessibility Services, a confidential service designed to support graduate and undergraduate students through their academic program. With the appropriate documentation, the student will work with both SAS and their graduate programs (normally their Graduate Chair and/or Course instructor) to ensure that appropriate academic accommodations to program requirements are arranged. These accommodations include

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individual counselling, alternative formatted literature, accessible campus transportation, learning strategy instruction, writing exams and assistive technology instruction.

**Online Teaching Provision:** In the event of a COVID-19 resurgence during the course that necessitates the course delivery moving away from face-to-face interaction, all remaining course content will be delivered entirely online, either synchronously (i.e., at the times indicated in the timetable) or asynchronously (e.g., posted on OWL for students to view at their convenience). The grading scheme will not change. Any remaining assessments will also be conducted online as determined by the course instructor.

In the event of a health lockdown, tests and examinations in this course will be conducted using a remote proctoring service. By taking this course, you are consenting to the use of this software and acknowledge that you will be required to provide personal information (including some biometric data) and the session will be recorded. Completion of this course will require you to have a reliable internet connection and a device that meets the technical requirements for this service. More information about this remote proctoring service, including technical requirements, is available on Western's Remote Proctoring website at <https://remoteproctoring.uwo.ca>.



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