COVID-19 REVISED: March 16, 2020

Math 9406B: Pro-finite Groups (Winter 2020)

Essential information

- **Lectures:** Tu 11:30-12:30 and Th 10:30-12:30 in MC 108
- **Office hours:** by appointment
- **Instructor:** David Riley
- **Email:** dmriley (at) uwo.ca
- **Office:** MC 136
- **Website:** on OWL
- **Prerequisites:** one course each in group theory, ring theory and general topology. Basic knowledge of Lie algebra would be helpful, but is not required.

What are pro-finite groups?

Let p be a prime number. A pro-p group is a topological group that is the projective limit of a directed system of finite p-groups. Pro-p groups belong to the larger class of pro-finite groups, which are projective limits of arbitrary finite groups. The theory of pro-p groups is the most explored part of the theory of pro-finite groups and the most useful in its applications. The impetus for the study of pro-finite and pro-p groups comes from at least three directions. First, in algebraic number theory and arithmetic algebraic geometry, absolute Galois groups and more generally étale fundamental groups of schemes are, by construction, pro-finite groups. Recent progress in Galois theory both suggest and use developments in the theory of pro-finite groups -- primarily pro-p groups. Second, pro-finite groups serve as a new approach for the study of infinite discrete groups, as every group has a pro-finite completion, whose properties may be used to understand the original group. Third, and more recently, pro-p groups have been used to study finite p-groups, in particular for the frontier problem of finding a possible classification of finite p-groups, analogous to the famous Classification of Finite Simple Groups.

Textbook

We will largely follow *Analytic Pro-p Groups*, 2nd edition, Cambridge University Press by J. Dixon, M. du Sautoy, A. Mann and D. Segal. There is no need to buy this book. Other references will be added later.

Course content

As time permits, the main topics will include:

- Basic background in groups, topology, Lie algebras and p-adic numbers
- Pro-finite and pro-p groups
- Powerful pro-p groups
- Pro-p groups of finite rank
- Uniformly powerful pro-p groups
- Pro-p groups of finite class
- Dimension subgroup methods
- Associated graded algebras
- Restricted Lie algebras

Original Evaluation

The final grades will be based on the following components:

- four assignments: 60%
- project: 20%
- presentation: 20%

REVISED Evaluation

The final grades will be based on the following components:

- four assignments: 60%

- project: 30%
- on-line participation: 10%

Assignments

There will be **four assignments**, each worth 15% of the final grade.

Project/Presentation

Each student will write a **10-15 page long document** in LaTeX, surveying a topic in pro-finite group theory. The instructor will prepare a list of potential projects, but students are encouraged to suggest their own topics. Associated to the project, each student will give a **50-minute presentation** during the last two weeks of the course. The presentations cannot use slides, must include a theorem, and a proof of at least one lemma.

Accommodation and Accessibility

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or supporting documentation to the Academic Counselling Office of your home faculty as soon as possible. If you are a Science student, the Academic Counselling Office of the Faculty of Science is located in WSC 140, and can be contacted at scibmsac@uwo.ca. For further information, please consult the university's medical illness policy at http://www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_medical.pdf.

If you miss the Final Exam, please contact your faculty's Academic Counselling Office as soon as you are able to do so. They will assess your eligibility to write the Special Exam (the name given by the university to a make-up Final Exam). You may also be eligible to write the Special Exam if you are in a "Multiple Exam Situation". http://www.registrar.uwo.ca/examinations/exam_schedule.html

Academic Policies

The website for Registrarial Services is http://www.registrar.uwo.ca.

In accordance with policy, http://www.uwo.ca/its/identity/activatenonstudent.html, the centrally administered e-mail account provided to students will be considered the individual's official university e-mail address. It is the responsibility of the account holder to ensure that e-mail received from the University at his/her official university address is attended to in a timely manner.

Scholastic Offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at this website: http://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_undergrad.pdf

Student Accessibility Services

Western is committed to achieving barrier-free accessibility for all its members, including graduate students. As part of this commitment, Western provides a variety of services devoted to promoting, advocating, and accommodating persons with disabilities in their respective graduate program.

Graduate students with disabilities (for example, chronic illnesses, mental health conditions, mobility impairments) are encouraged to register with Student Accessibility Services, a confidential service designed to support graduate and undergraduate students through their academic program. With the appropriate documentation, the student will work with both SAS and their graduate programs (normally their Graduate Chair and/or Course instructor) to ensure that appropriate academic accommodations to program requirements are arranged. These accommodations include individual counselling, alternative formatted literature, accessible campus transportation, learning strategy instruction, writing exams and assistive technology instruction. http://www.sdc.uwo.ca/ssd/